Enrollment No:-__________ Exam Seat No:-________

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: 5SC04ACC1 **Subject Name: Advanced Complex Analysis**

Course Name: M.Sc. (Mathematics) Date: 19/5/2015

Semester: IV Marks: 70

Time: 10:30 TO 01:30

Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION-I

Q-1 a) Define index of γ. (02)

b) State open mapping theorem. (02)

c) Define zero of function of multiplicity m. (02)

d) State Morera's theorem. (01)

Q-2 a) Let $\gamma: [0,1] \to \mathbb{C}$ be a closed rectifiable curve. Suppose $a \notin \{\gamma\}$ then prove that $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is an integer. (07)

b) Let G be a region and $f: G \to \mathbb{C}$ is analytic function on G. Suppose $a_1, a_2, \dots, a_m$ are the zero's of f in G. Suppose γ is a closed rectifiable curve on G such that γ does not pass through any zero and $n(\gamma, w) = 0\ \forall\ w \in \mathbb{C} - G$. Then prove that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}\,dz = \sum_{k=1}^{m} n(\gamma, a_k)$ (07)

OR

Q-2 a) Let γ and σ are two rectifiable paths in $\mathbb{C}$ with $\gamma(1) = \sigma(0)$ then prove that $n(\gamma + \sigma, a) = n(\gamma, a) + n(\sigma, a)$ where $a \notin \{\gamma\} \cup \{\sigma\}$. (07)

b) Let γ be a rectifiable curve. Suppose $\emptyset$ is a complex valued function defined on $\{\gamma\}$. If $f_m(z) = \int_\gamma \frac{\emptyset(w)dw}{(w-z)^m}\,dw, z \notin \{\gamma\}$ for $m \geq 1$ then prove that f_m's are analytic on $G = \mathbb{C} - \{\gamma\}$ and $f_m{}'(z) = m\, f_{m+1}(z)$. (07)

Q-3 a) State and prove Argument Principal. (07)

b) Let G be an open set in $\mathbb{C}$ and suppose $f: G \to G$ be analytic. Let γ be a closed rectifiable curve in G such that $n(\gamma, w) = 0 \forall\ w \in \mathbb{C} - G$. Then for $a \in G - \{\gamma\}$ then prove that $\frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-a}\,dz = n(\gamma, a)\, f(a)$. (07)

OR

Q-3	a)	Let f be analytic within and on the circle $\{z: \lvert z-a\rvert < R\}$. Then prove that $f'(a) = \frac{1}{\pi r}\int_0^{2\pi} Re\ f\left(a + r\ e^{i\theta}\right) e^{-i\theta}\, d\theta,\ 0 < r < R.$	(07)
	b)	State and prove Arzela – Ascoli's theorem.	(07)

SECTION-II

Q-4	a)	Let f be an entire function and $f(z) = u + iv$. Suppose $u = Re\ f(z)$ is bounded. Then prove that f reduce to a constant.	(02)
	b)	State Reimann mapping theorem.	(02)
	c)	State Montel's theorem.	(02)
	d)	Define convex function.	(01)
Q-5	a)	Suppose f and g be meromorphic functions on a neighbourhood of $\bar{B}(a, R)$ which have no zeros or poles on the circle $\gamma = \{z: \lvert z-a\rvert = R\}$. Let Z_f, Z_g, P_f and P_g are the number of zeros and poles of f and g lying inside γ counted according to their multiplicity and if $\lvert f(z) + g(z)\rvert < \lvert f(z)\rvert + \lvert g(z)\rvert$ for z on γ. Then prove that $Z_f - P_f = Z_g - P_g$.	(07)
	b)	Prove that a function f is convex on (a,b) if and only if for each triplet points s, t, u with $s \le t \le u \frac{f(t)-f(s)}{t-s} \le \frac{f(u)-f(t)}{u-t},\ a < s < t < u < b.$	(07)

OR

Q-5	a)	Using Rouche's theorem prove fundamental theorem of algebra.	(07)
	b)	A differentiable function f is convex if and only if f' is increasing.	(07)
Q-6	a)	Suppose $Re\ z > 0\ \forall\ k \ge 1$ then $\prod_{k=1}^{\infty} z_k$ is convergent if and only if $\sum_{k\ge1} \log z_k$ is convergent.	(07)
	b)	Let f be analytic within and on disk $\lvert z\rvert \le \rho$. Suppose $0 < r < R < \rho$ then prove that $f\left(r\ e^{i\theta}\right) = \frac{1}{2\pi}\int_0^{2\pi} \frac{(R^2-r^2) f(Re^{i\phi})}{R^2 - 2Rr\cos(\theta-\phi) + r^2}\, d\phi.$	(07)

OR

Q-6	a)	If all zero's of a polynomial $p(z)$ lies in a half plane then prove that the zero's of $p'(z)$ also lies in the same half plane.	(07)
	b)	State and prove Weirstrass factorization theorem.	(07)